Variational Methods for Computing Non-Local Quantum Strategies

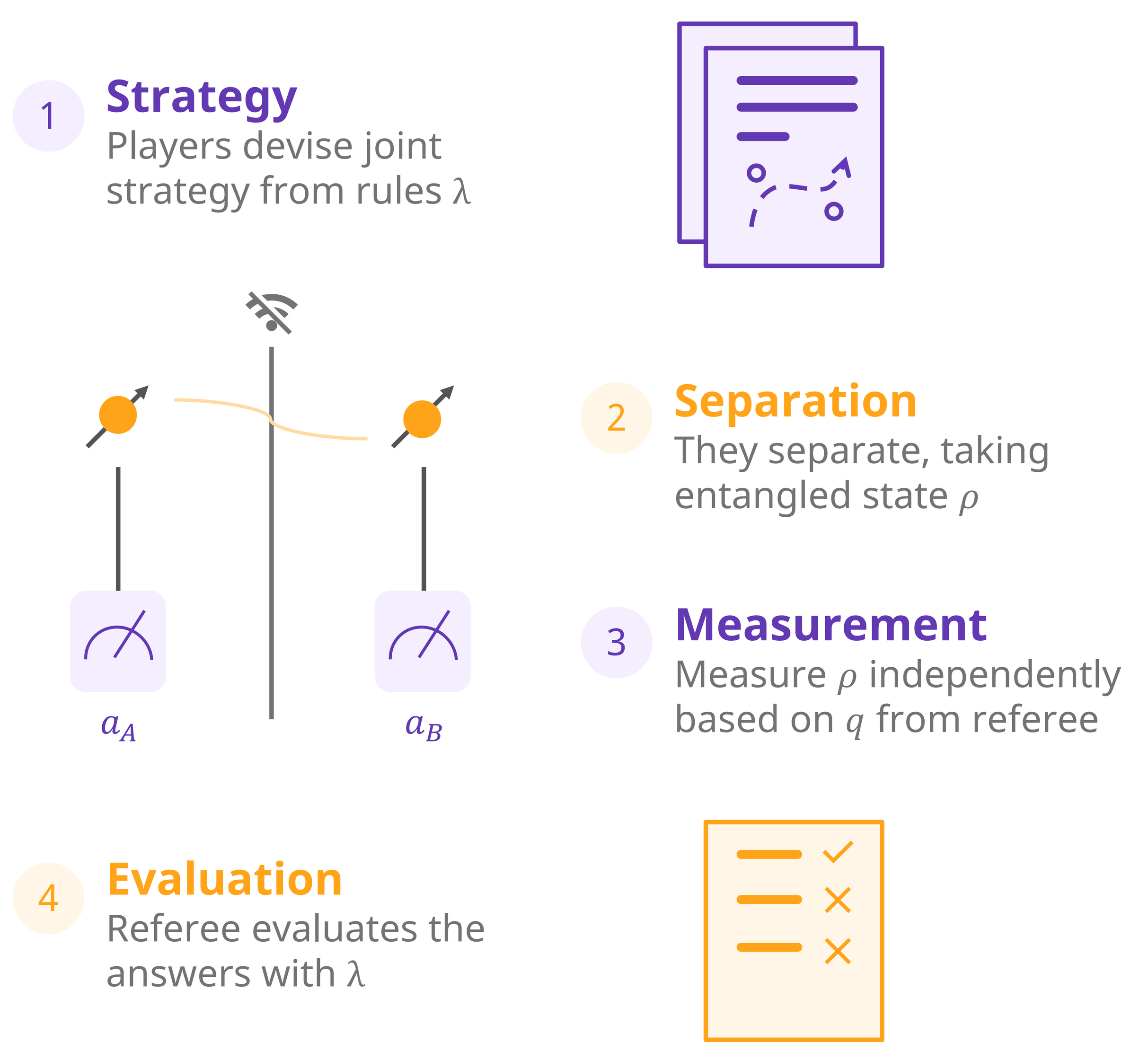
Jim Furches[1]
Nathan Wiebe[2 3 4]
Carlos Ortiz Marrero[5 6]
[1]VT [2]UofT ([3]HPC/[5]AIDA) PNNL [4]CIFAR [6]NCSU

Introduction

Non-local games explore the boundaries between classical physics, quantum theory, and other non-signaling theories[1 2]. Broad classes of games with a probable quantum advantage have been revealed[3]. However, constructing optimal quantum strategies for non-local games remains a challenge.

Non-Local Games

Quantum strategies for non-local games leverage **quantum entanglement as a resource**. Each game proceeds as follows



The value of the game is

$$V(G) = \sum_{qa} \lambda(a|q) p(a|q) p(q) .$$

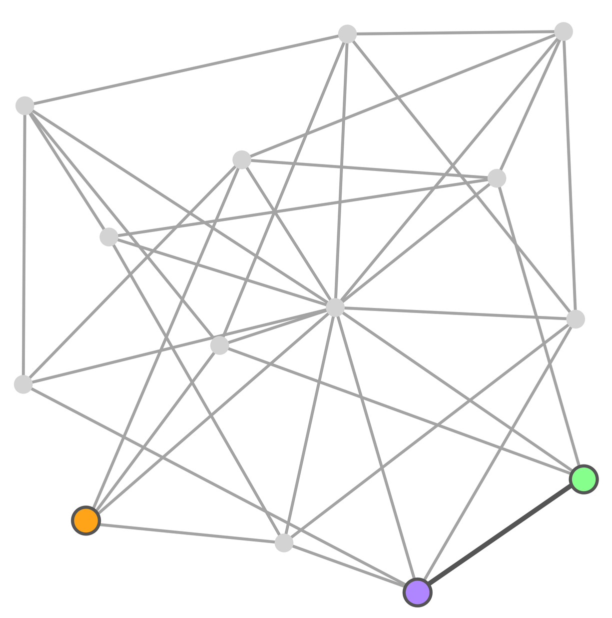
For a quantum strategy,

$$p(a|q) = \mathrm{Tr}[\rho \mathcal{M}_{a|q}]$$

$$\mathcal{M}_{a|q} = \otimes_i \mathcal{M}_{a_i|q_i} \qquad \text{(non-signaling)}$$

Quantum Chromatic Game

The quantum chromatic game[4] is derived from the **graph coloring problem**. Players must properly color a vertex or an edge.



G_{14} Graph[4]
Classical $\chi(G) = 5$
Quantum $\chi_q(G) = 4$

Questions: 88
Answers: 4 (χ_q)

Vertex v
Player colors must match
$c_A = c_B$

Edge $e = (v_A, v_B)$
Player colors must differ
$c_A \neq c_B$

$$H = -\frac{1}{|Q|}\left[\sum_v U_v^\dagger P_{cc} U_v + \sum_e U_e^\dagger (I - P_{cc}) U_e\right]$$

$$\mathrm{Tr}[\rho H] = -V(G)$$

$$\mathrm{Tr}[\rho P_{cc}] = p(c_A = c_B)$$

U_v and U_e are the measurement layers. A perfect ($V(G) = 1$) strategy is known to exist[4] with constraints on the measurements

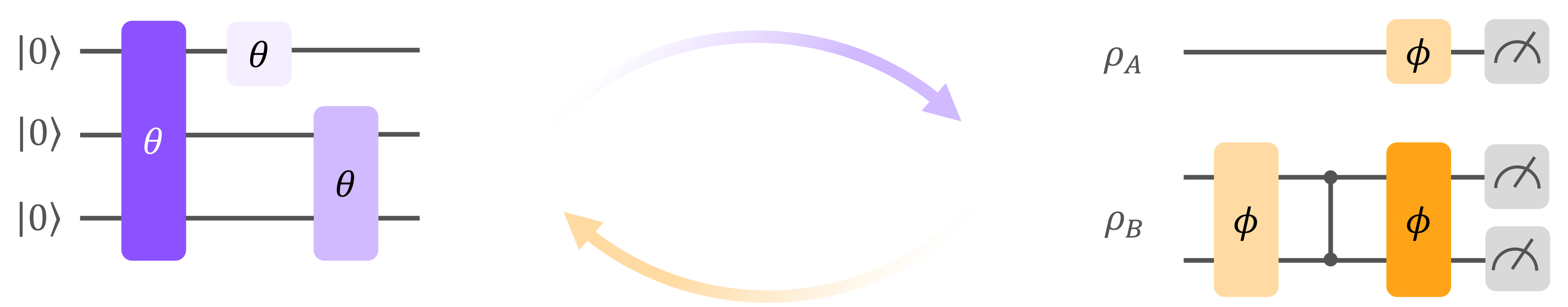
$$\mathcal{M}_{a_A|q_A} = \mathcal{M}^*_{a_B|q_B}$$

Dual-Phase Optimization

- Custom variational algorithm for computing quantum strategies
- **ADAPT-VQE**[5] for state preparation creates compact circuits for NISQ hardware.

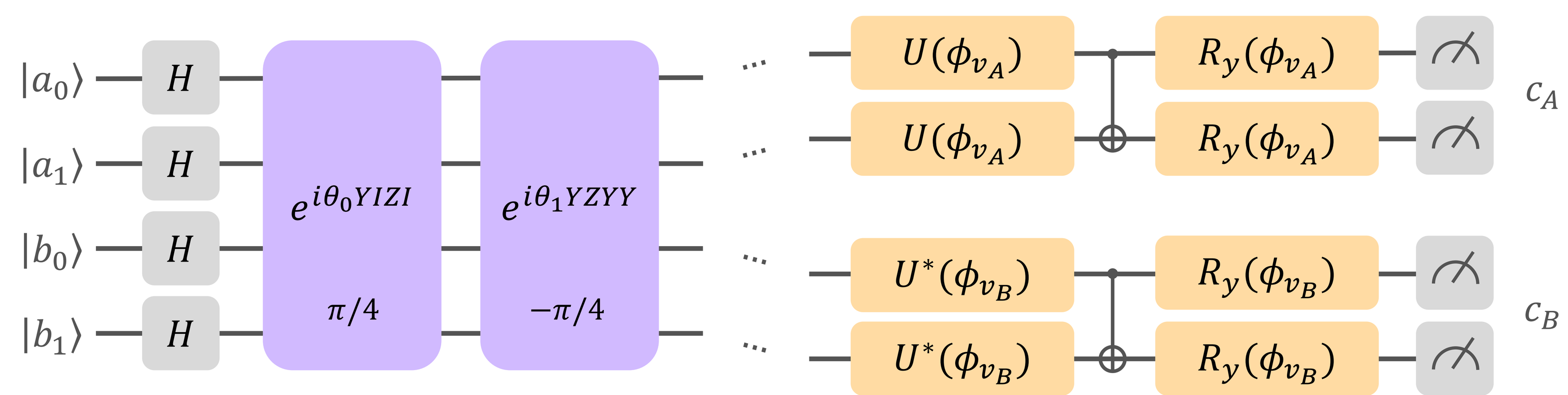
1 **State Preparation**
Use VQE to prepare $\rho(\theta)$ for $H(\phi)$

2 **Tune Measurements**
Perform grad. descent on parameters ϕ



Results

- A perfect quantum G_{14} strategy was created by 500 randomized DPO trials **(classical)**.
- The strategy was executed on 11 IBM quantum devices, 88 circuits per device **(quantum)**.
- We observed the strategy has properties desirable for benchmarking and self-testing.



Shared State $U(\theta)$
$|\psi\rangle = \frac{1}{2} H^{\otimes 4} \sum_c |cc\rangle$

U_3Ry Layer $U_q(\phi)$
ϕ has 112 params

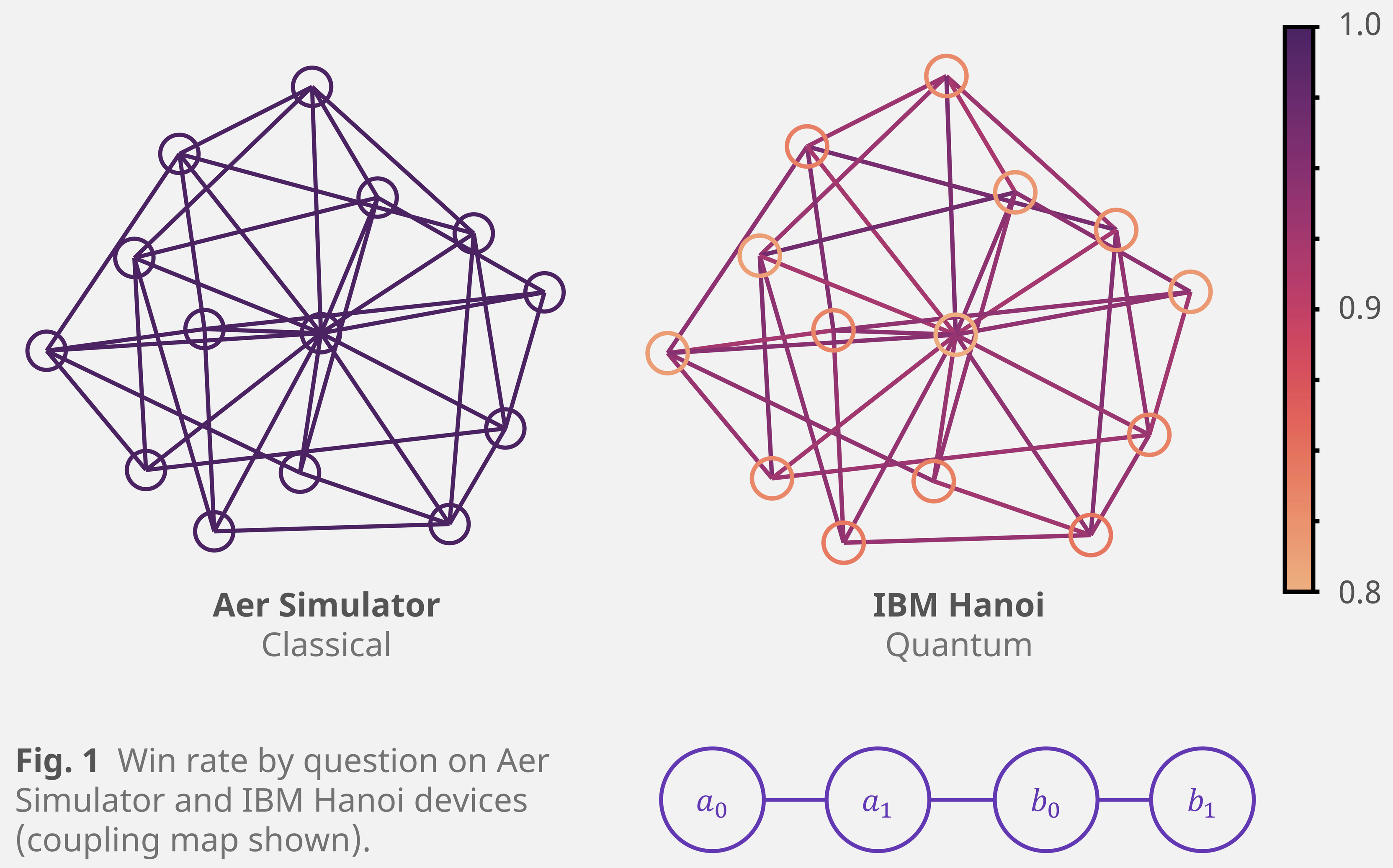


Fig. 1 Win rate by question on Aer Simulator and IBM Hanoi devices (coupling map shown).

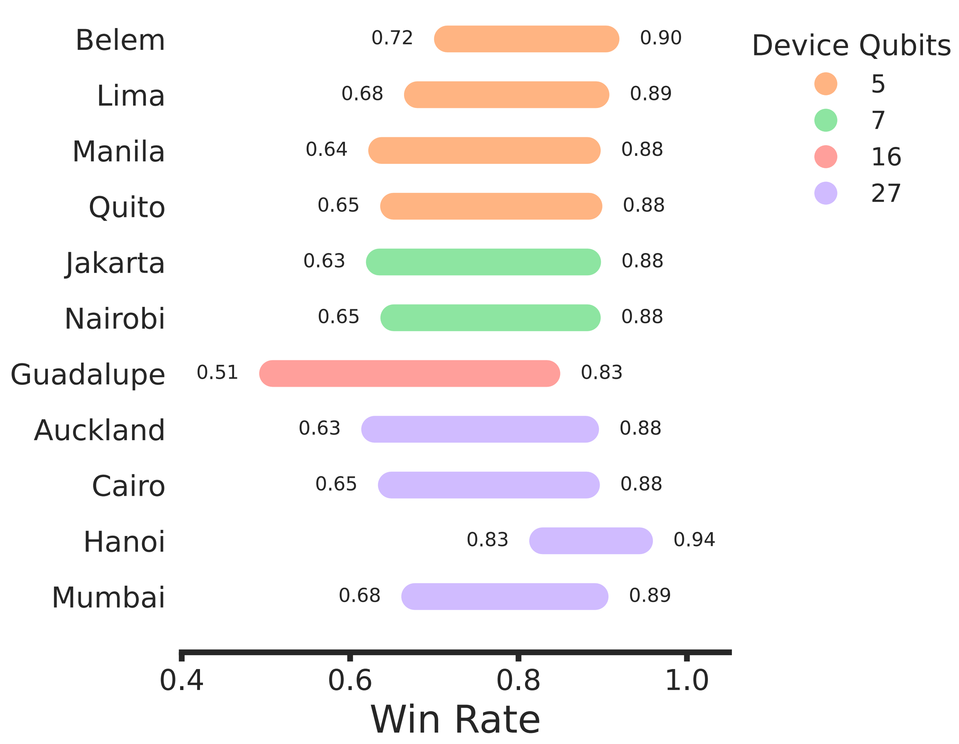


Fig. 2 Vertex-edge win rate on each quantum device, colored by the number of available physical qubits. The circuit was executed on 4 physical qubits.

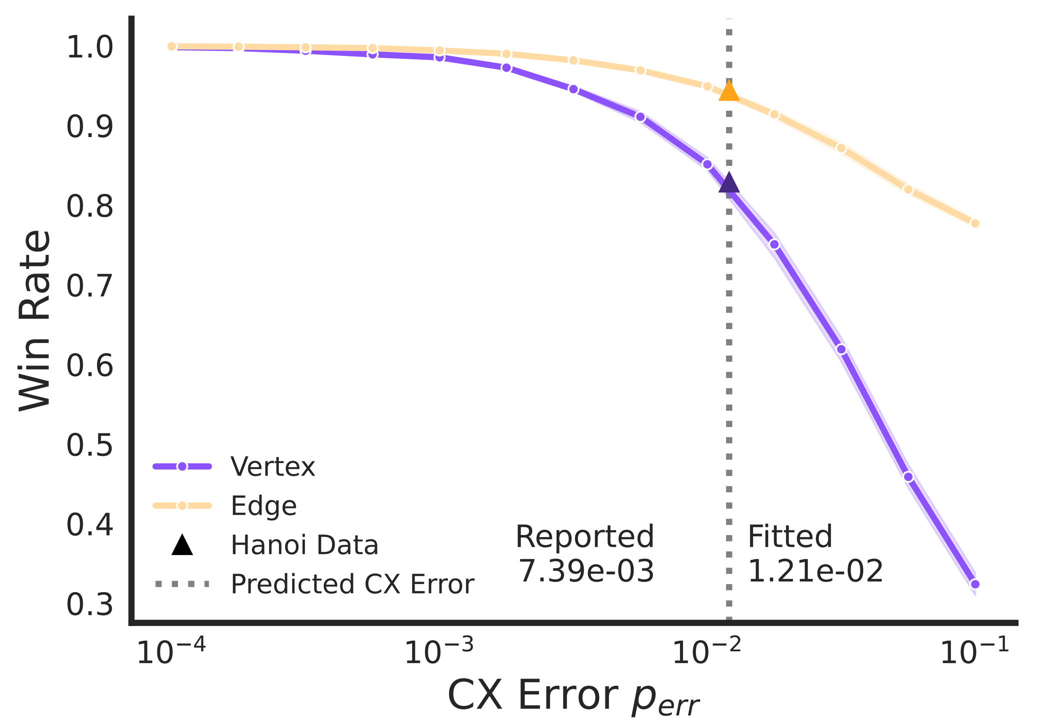


Fig. 3 Win rate of strategy classically simulated on Hanoi coupling map, with Pauli noise added to CX gates with probability p_{err}.

- **Vertex** questions are sensitive to noise **(benchmarking)**.
- **Edge** questions prove a device uses quantum resources if **>97% (self-testing)**.

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Acknowledgements

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